Lecture 1
Special relativity: applications to astrophysics 2

Einstein’s new relativity
- Galilean:
  - The laws of mechanics are the same in all inertial frames of reference
  - time and space are the same in all inertial frames of reference
- Einstein:
  - The laws of physics are the same in all inertial frames of reference
  - the speed of light in the vacuum is the same in all inertial frames of reference
  - time spans and distances are relative

Consequences
- breakdown of simultaneity:
  - whether or not two events happen simultaneously depends on the speed at which you are moving with respect to the events
- time dilation
  \[ \Delta t = \gamma \Delta t_0 \]
- length contraction
  \[ L = L_0 / \gamma \]

Boost factor
- \[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]
- **Boost factor**
- **time dilation**: \[ \Delta t = \gamma \Delta t_0 \]
- **length contraction**: \[ L = L_0 / \gamma \]
  - \( v = 0.1c \Rightarrow \gamma = 1.005 \Rightarrow 0.5\% \) boost
  - \( v = 0.5c \Rightarrow \gamma = 1.155 \Rightarrow 15.5\% \) boost
  - \( v = 0.9c \Rightarrow \gamma = 2.294 \Rightarrow 129.4\% \) boost
  - \( v = 0.999c \Rightarrow \gamma = 22.37 \)

This is messy, so let’s clean up a bit
- proper time: time measured by a clock at rest with respect to a specific observer i.e. clock at the fastest possible rate
- proper length: length of an object as measured in its own rest frame i.e. largest possible length
- time and length in other inertial frames can be calculated by the so-called Lorentz transformation (i.e. multiplying with or dividing by the boost factor)

So \( c + c = c \)? yes, sort of ...
- Q: So how do we add velocities?
  - A: by properly applying Lorentz transforms:
    \[ v_{\text{tot}} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \]
  - Example: shoot a torpedo (\( v_t = 0.5c \)) from the Enterprise, moving at \( v_e = 0.5c \)
    outside observer: \( v_{\text{tortago}} = 0.8c \)

Doppler effect (for sound)
The pitch of an approaching car is higher than that of a car moving away.

Doppler effect (for light)
The light of an approaching source is shifted to the blue, the light of a receding source is shifted to the red.

Doppler effect
The light of an approaching source is shifted to the blue, the light of a receding source is shifted to the red.

Doppler effect
redshift:
\[ 1+z = \frac{1+v/c}{1-v/c} \]
z=0: not moving
z=2; v=0.8c
z=10; v=c

The amount of spectral shift tells us the velocity of the object:
\[ \frac{\Delta \lambda}{\lambda} = \frac{v}{c} \]

Relativistic headlight effect
Beaming
Example GRB beaming

\[ \cos \theta = \frac{\cos \theta - \beta}{1 - \beta \cos \theta} \]

\[ \sin \theta = \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta} \]
Doppler measurements of extra-galactic distances

\[ v = H_0 D \]
\[ v = c (\Delta \lambda / \lambda_0) = cz \]
\[ d = \frac{cz}{H_0} \]

Only valid for small \( z \)!

Minkowski’s spacetime

- As we have seen, time intervals, lengths, and simultaneity is relative and depend on the relative velocity of the observer.
- Velocity connects time and space
- Let’s stop separating space and time, let’s rather talk about spacetime
- Spacetime is 4 dimensional, 3 spatial + 1 time dimension but is space and time really the same thing?

Minkowski diagram

**World lines — slowly moving**

**World lines — fast moving**

**Faster than speed of light?**

World line of a particle

World line of a particle
World lines — accelerated
\[ c \times t \]

World lines — decelerated
\[ c \times t \]

geometrical interval
\[ \Delta s = \sqrt{\Delta y^2 + \Delta x^2} \]

Spacetime interval
\[ \Delta s^2 = (c \Delta t)^2 - \Delta x^2 \]

- sign: difference between space and time
- \( \Delta s^2 \) is invariant under Lorentz transformation
- for particle moving at speed of light:
  \[ \Delta x = c \Delta t \quad \Rightarrow \quad \Delta s^2 = 0 \]
  \( \Rightarrow \) light like (null) distance

Character of spacetime intervals
- \( \Delta s^2 > 0 \quad \Rightarrow \quad c \Delta t > \Delta x \)
  - spatial distance can be traveled by speed of light
  - there exist an inertial frame, in which the two events happen at the same position
  - but they never happen simultaneously
  \( \Rightarrow \) time like distance
- \( \Delta s^2 < 0 \quad \Rightarrow \quad c \Delta t < \Delta x \)
  - spatial distance cannot be traveled by speed of light
  - there exist an inertial frame, in which the two events happen simultaneously
  - but they never happen at the same place
  \( \Rightarrow \) space like distance
Future, past, and elsewhere

Future

Past

elsewhere

elsewhere

Δs² > 0

Δs² < 0

Principle of causality

- All observers agree that B is in the past of A and C is in the future
- Some see A happen first, some see D happen first
- Cause must always precede the effect
- A must not influence D and vice versa
- nothing can move faster than speed of light

Future, past, and elsewhere

Future

Past

elsewhere

elsewhere

Δs² > 0

Δs² < 0